Разбор КР 1

1) $f = \dfrac{sh(\cos(\sqrt[3]{x^2+1}))}{\log(2x+e)}$, $f'(x) = \dfrac{\cosh(\cos(\sqrt[3]{x^2+1}))\cdot(-\sin(\sqrt[3]{x^2+1}))\cdot\frac{1}{3}(x^2+1)^{-\frac{2}{3}}\cdot 2x}{\log(2x+e)}$

$+ sh(\cos\sqrt[3]{x^2+1})\left(\dfrac{-1}{\log^2(2x+e)}\right)\cdot\dfrac{1}{2x+e}\cdot 2$

$f'(0) = \dfrac{\cosh(\cos(1))\cdot(-\sin 1)\cdot\frac{2}{3}\cdot 1\cdot 0}{\log e} + sh(\cos 1)\left(-\dfrac{1}{\log^2 e}\right)\cdot\dfrac{2}{e} =$

$= sh\cos 1\left(-\dfrac{2}{e}\right) = -\dfrac{2\, sh\cos 1}{e}$

2) $C_{2n}^{n} < 2^{2n-2}$ при $n \geq 5$?

База: $C_{10}^{5} < 2^{10-2}$? $\Leftrightarrow \dfrac{10!}{((5)!)^2} \overset{?}{<} 2^8 \Leftrightarrow \dfrac{6\cdot 7\cdot 8\cdot 9\cdot 10}{2\cdot 3\cdot 4\cdot 5} < 2^8$

$\Leftrightarrow \dfrac{\cancel{3}\cdot 7\cdot 9\cdot \cancel{8}}{2\cdot\cancel{3}\cdot 4\cancel{5}} < 2^3 \Leftrightarrow 7\cdot 9 < 16$ – ok.

Переход: $C_{2(n+1)}^{(n+1)} \overset{?}{<} 2^{2(n+1)-2} \Leftrightarrow \dfrac{(2n+2)!}{((n+1)!)^2} \overset{?}{<} 4\cdot 2^{2n-2} \Leftrightarrow \dfrac{(2n+2)(2n+1)}{(n+1)^2}\cdot C_{2n}^{n} \overset{?}{<}$

$\overset{?}{<} 4\, 2^{2n-2} \Leftarrow \dfrac{(2n+2)(2n+1)}{(n+1)^2} < 4 \Leftrightarrow \dfrac{2(2n+1)}{n+1} < 4 \Leftrightarrow \dfrac{2n+1}{n+1} < 2 \Leftrightarrow$

$\Leftrightarrow 2n+1 < 2n+2$ – ok

3) $\underbrace{\sin x + tg x}_{f} \geq \underbrace{2x}_{g}$, $x \in (0, \frac{\pi}{2})$; $f(0) = g(0) = 0$,

$f' = \cos x + \dfrac{1}{\cos^2 x} \geq \cos x + \dfrac{1}{\cos x} \geq 2 = g'(x) \Rightarrow$ ok

$\left(x + \dfrac{1}{x} \geq 2 \;\; \forall x \in (0,\infty)\right.$ т.к. $\left. x^2 - 2x + 1 = (x-1)^2 \geq 0\right)$

4) $\lim \underbrace{n\, tg\frac{1}{n}}_{\overset{\shortparallel}{\frac{n\sin\frac{1}{n}}{\cos\frac{1}{n}}} \sim\, n\sin\frac{1}{n} \to 1} \underbrace{\sqrt{4+2^{-n}}}_{\to 2} = 2$, ищем $N(\varepsilon)$

$\left|n\, tg\frac{1}{n}\sqrt{4+2^{-n}} - 2\right| < \varepsilon \Leftarrow \left|n\, tg\frac{1}{n} - 1\right|\left|\sqrt{4+2^{-n}}\right| + \left|\sqrt{4+2^{-n}} - 2\right| < \varepsilon$

$\Leftarrow 4\left|n\, tg\frac{1}{n} - 1\right| + \dfrac{|4+2^{-n} - 4|}{\sqrt{4+2^{-n}}+2} < \varepsilon \Leftarrow 4\left|\dfrac{tg\frac{1}{n} - tg 0}{\frac{1}{n} - 0} - 1\right| + 2^{-n} < \varepsilon$

$$\Leftarrow \left|\frac{1}{\cos^2 x_n} - 1\right| + 2^{-n} < \varepsilon \Leftarrow \frac{\sin^2 x_n}{\cos^2 x_n} + 2^{-n} < \varepsilon \Leftarrow \left[\begin{array}{l}\cos^2 x_n \geq \cos^2\frac{1}{n}\\ \sin^2 x_n \leq \frac{1}{n^2}\end{array}\right]$$

т. Лагранжа, $x_n \in \left[0, \frac{1}{n}\right]$

$$\Leftarrow \frac{1}{n^2}\cdot\frac{1}{\cos^2\frac{1}{n}} + 2^{-n} < \varepsilon \Leftarrow \left[\cos^2\frac{1}{n} > \cos 1\right] \Leftarrow \frac{1}{n^2\cos 1} + 2^{-n} < \varepsilon$$

$\Leftarrow$ [нер-во Бернулли] $\Leftarrow \frac{1}{n^2\cos 1} + \frac{1}{n} < \varepsilon \Leftarrow$

$\Leftarrow \frac{1}{n}\left(\frac{1}{\cos 1} + 1\right) < \varepsilon$, $N(\varepsilon) := \left[\left(\frac{1}{\cos 1} + 1\right)\frac{1}{\varepsilon}\right]$

5) $e^{x^2\cos^2 x}$ до $o(x^8)$: $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + o(x^6)$

$$\cos^2 x = \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} + o(x^6)\right)^2 = 1 + \frac{x^4}{(2!)^2} + \frac{(-2x^6)}{2!\cdot 4!} + \frac{2x^4}{4!} + \frac{(-2x^2)}{2!} + o(x^6)$$

$$x^2\cos^2 x = x^2 - x^4 + \left(\frac{1}{4} + \frac{1}{12}\right)x^6 + o(x^8)$$
$$= x^2 - x^4 + \frac{x^6}{3} + o(x^8)$$

$$e^{x^2\cos^2 x} = 1 + \left(x^2 - x^4 + \frac{x^6}{3} + o(x^8)\right) + \frac{\left(x^2 - x^4 + \frac{x^6}{3} + o(x^8)\right)^2}{2!} + \frac{x^6}{3!} + o(x^8)$$
$$= 1 + x^2 - x^4 + \frac{x^6}{3} + \frac{x^4}{2!} - \frac{2x^6}{2!} + \frac{x^6}{3!} + o(x^8)$$
$$= 1 + x^2 + x^4\left(-1 + \frac{1}{2}\right) + x^6\left(\frac{1}{3} - 1 + \frac{1}{6}\right) + o(x^8)$$
$$= 1 + x^2 - \frac{x^4}{2} - \frac{x^6}{2} + o(x^8)$$

$\tilde{x} = \frac{x}{2}$

6) $\lim \left(sh\left(\sqrt{4+x^2} - 2\right) - \sin\left(\sqrt{4+x^2} - 2\right)\right)\cdot\left(2 + (4x)^{-3}\right)^2 =$

$$\sqrt{4+x^2} - 2 = 2\left(\sqrt{1+\frac{x^2}{4}} - 1\right) = 2\left(\frac{\tilde{x}^2}{2} - \frac{\tilde{x}^4}{8} + \frac{\tilde{x}^6}{16}\right) + o(\tilde{x}^8) = \tilde{x}^2 - \frac{\tilde{x}^4}{4} + \frac{\tilde{x}^6}{3} + o(\tilde{x}^8)$$

$$sh\, x = \frac{e^x - e^{-x}}{2} = \frac{\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^4)\right) - \left(1 - x + \frac{x^2}{2} - \frac{x^3}{6} + o(x^4)\right)}{2} = x + \frac{x^3}{6} + o(x^5)$$

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$\cancel{sh(\sqrt{4+x^2}-2) = \left(x^2 - \frac{x^4}{32} + \frac{x^6}{8} + o(x^8)\right)^2}$

$$\sin x = x - \frac{x^3}{3!} + o(x^5) \Rightarrow sh\, x - \sin x = \frac{2x^3}{6} + o(x^5) = \frac{x^3}{3} + o(x^5)$$

$$sh\left(\sqrt{4+x^2} - 2\right) - \sin\left(\sqrt{4+x^2} - 2\right) = \frac{\left(\sqrt{4+x^2} - 2\right)^3}{3} + o\left(\left(\sqrt{4+x^2} - 2\right)^5\right) =$$
$$= \frac{\tilde{x}^6}{3} + o(x^{10})$$

$$= \lim \frac{\tilde{x}^6}{3}\left(2 + \frac{1}{4^3x^3}\right)^2 = \frac{1}{3\cdot 2^{12}\cdot 2^6} = \frac{1}{3\cdot 2^{18}}$$