Spinor gravity on lattice

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The essence of GR is invariance under diffeomorphisms (general coordinate transformations)

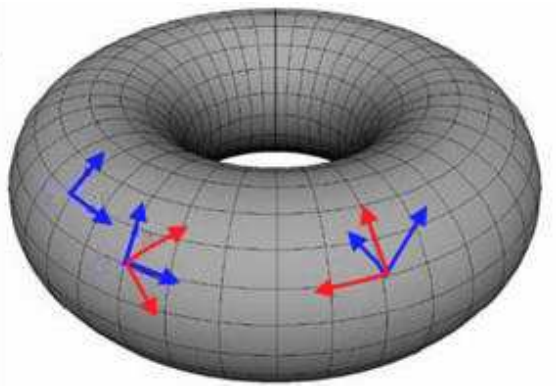
$$x^\mu \to x'^\mu(x), \qquad A_\mu(x) \to A_\nu(x')\frac{dx'^\nu}{dx^\mu}, \qquad ..., \qquad d^4x \to \left\| \frac{dx}{dx'} \right\| d^4x'$$

General Relativity in terms of the metric tensor

$$S = -\frac{M_P^2}{16\pi}\int d^4x \left(\underbrace{2\Lambda\sqrt{g}}_{\text{cosmological term}} + \underbrace{\sqrt{g}\bar{R}}_{\text{Einstein–Hilbert term}} \right),$$

$$\Lambda \simeq 8.65 \cdot 10^{-66}\text{eV}^2, \qquad M_P = 1.22 \cdot 10^{19}\text{GeV}, \qquad \frac{\Lambda}{M_P^2} \simeq 5.82 \cdot 10^{-122}$$

Tetrad field



It is more convenient to use Einstein-Cartan formulation.
$e_\mu^A(x)$ is tetrad (vierbein, or frame field):

$$g_{\mu\nu}(x) = e_\mu^A e_\nu^A, \qquad e_A^\mu = (e_\mu^A)^{-1}, \qquad A = 1,..,d;\ \mu = 1,..,d.$$

The indices $A, B, C, ..$ are "flat" Lorentz indices (we do not care about lower and upper Lorentz indices).

First order formalism (Einstein-Cartan)

Local Lorentz invariance

Ortho-normal coordinate systems can be independently rotated at every point (Gauge Lorentz symmetry)

$$e^A_\mu(x) \to O^{AB} e^B_\mu(x), \qquad O \in \text{Lorentz group}$$

$$\partial_\mu T^A \to D^{AB}_\mu T^B = \left(\delta^{AB}\partial_\mu + \omega^{AB}_\mu\right) T^B$$

$\omega^{AB}_\mu(x) = -\omega^{BA}_\mu$, spin-connection, (AB) enumerates $\frac{d(d-1)}{2}$ generators of the Lorentz group.
Historically, it is the first example of Yang-Mills theory,[Cartan,22][Fock,28]

Einstein-Cartan gravity

$$S = \frac{M_P^2}{16\pi}\int d^4x\Big(\underbrace{-2\Lambda \det e}_{\Lambda\sqrt{g}} - \frac{1}{4}\underbrace{\epsilon^{\mu\nu\alpha\beta}\epsilon_{ABCD}\mathcal{F}^{AB}_{\mu\nu}e^C_\alpha e^D_\beta}_{\sqrt{g}R} - \underbrace{\frac{\iota}{2}\epsilon^{\mu\nu\alpha\beta}\mathcal{F}^{AB}_{\mu\nu}e^A_\alpha e^B_\beta}_{\substack{\text{Holst action}\\ =0 \text{ in EH}}} + \ldots\Big)$$

$$\mathcal{F}^{AB}_{\mu\nu} = [D_\mu D_\nu]^{AB} = \partial_\mu \omega^{AB}_\nu - \partial_\nu \omega^{AB}_\mu + [\omega_\mu \omega_\nu]^{AB}$$

The Einstein-Cartan gravity contains more d.o.f's then Einstain-Hilbert gravity and, in particular, contains new geometry structure: torsion.

$$\frac{1}{2}\left(\Gamma^{\alpha}_{\mu\nu} - \Gamma^{\alpha}_{\nu\mu}\right) e^{A}_{\alpha} = T^{A}_{\mu\nu}.$$

Without sources the Cartan action is equivalent to the Einstein- Hilbert action.

The fermions are the natural source for the torsion field.

$$S = \frac{M_P^2}{16\pi}\int d^4x \left(-2\Lambda \det e - \frac{1}{4}\epsilon^{\mu\nu\alpha\beta}\epsilon_{ABCD}\mathcal{F}^{AB}_{\mu\nu}e^{C}_{\alpha}e^{D}_{\beta} + e^{\mu}_{A}\psi^{\dagger}\gamma^{A}D_{\mu}\psi\right)$$

With the fermions the torsion is not zero at saddle point:

$$T^{A}_{\mu\nu} \sim \left(\bar{\Psi}...\Psi\right) \qquad \text{bilinear fermion current}$$

However, 4-fermion interaction induced by torsion is suppressed as $1/M_P^2$ (at least), e.g. [Hehl,78], [Diakonov,Tumanov,AV,11]

Speaking about quantum gravity I will keep in mind Einstein-Cartan formulation which is generically posses two symmetries

- Diffeomorphisms (gen.coordinate transformations) $x^\mu \to x'^\mu(x)$
- Gauge Lorentz invariance $e^A_\mu(x) \to O^{AB}(x) e^B_\mu(x)$

Nowadays, there are quite a lot of models of Quantum Gravity

- Loop quantum gravity (spin-foams)
- Dynamical triangulation models
- Group field theories
- String theory based models
- ...(Only in Wikipedia one can find 10 more approaches)

Every model has its own weak and strong points, and most part of them looks very unusual on first glance.

Boundless action

$$Z_{\text{gravity}}[J] = \int [De\ D\omega] e^{-S[J]}$$

Any diffeomerphism invariant action is not sign-definite.

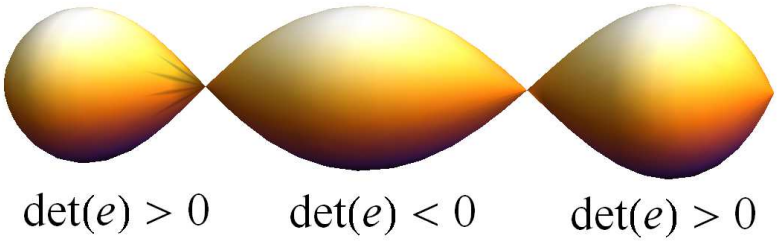
- The only index marring structure is (non sign-definite) Levy-Civita tensor. All the rest are fluctuating fields.
- Therefore, the action during fluctuations can contentiously passes through zero.

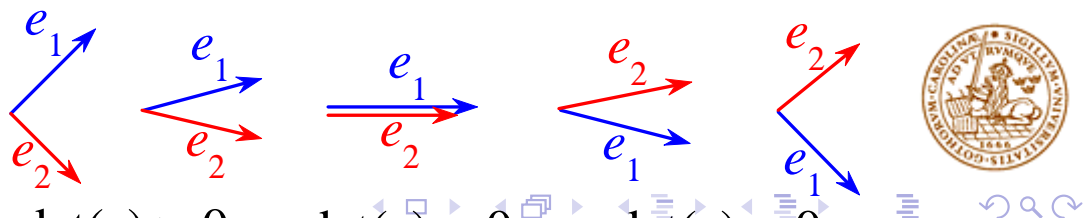
Large vacuum fluctuations are not suppressed, but even enforced.

The simplest example is cosmological term:

$$\int d^4x \det e = \int d^4x \epsilon^{\mu\nu\rho\sigma} \epsilon_{ABCD} e^A_\mu e^B_\nu e^C_\rho e^D_\sigma .$$

The "usual" quantum gravity is not-well defined in general, although the perturbative definition may exist (a-la phi^3 theory).





Spinor gravity

Making path integral well-define

- ◇ One of the ways to overcome the sign problem of gravity is to use only the compact and/or fermion degrees of freedom. The idea is not very new, see [Akama,78],[Volovik,90],[Wetterich,05-12].

$$e_\mu^A = \psi^\dagger \gamma^A D_\mu \psi - (D_\mu \psi)^\dagger \gamma^A \psi$$

This combination is Hermitian $e^\dagger = e$ and transforms as a 1-form and a Lorentz vector.

Another possibility

$$f_\mu^A = i(\psi^\dagger \gamma^A D_\mu \psi + (D_\mu \psi)^\dagger \gamma^A \psi) = i D_\mu (\psi^\dagger \gamma^A \psi)$$

Limitations on the action

The Grassman algebra gives some restrictions on the action:

- No tetrad with "upper" index: $\det(e)e^{\mu}_{A} = \epsilon^{\mu\nu\alpha\beta}\epsilon_{ABCD}e^{B}_{\nu}e^{C}_{\alpha}e^{D}_{\beta}$
- Tetrad in power $2^{\frac{d}{2}}$ is zero, $\left(e^{A}_{\mu}(x)\right)^{2^{d/2}} = 0.$

General Relativity action

- Cosmological term: $S_{cosm} = \int d^4x \frac{1}{4!}\epsilon_{ABCD}\epsilon^{\mu\nu\alpha\beta}e^{A}_{\mu}e^{B}_{\nu}e^{C}_{\alpha}e^{D}_{\beta}.$
- Einstein-Hilbert term: $S_{EH} = \int d^4x \frac{1}{12}\epsilon_{ABCD}\epsilon^{\mu\nu\alpha\beta}\mathcal{F}^{AB}_{\mu\nu}e^{C}_{\alpha}e^{D}_{\beta}$
- The only non-fermion term: $\int d^4x \frac{1}{4!}\epsilon_{ABCD}\epsilon^{\mu\nu\alpha\beta}\mathcal{F}^{AB}_{\mu\nu}\mathcal{F}^{CD}_{\alpha\beta} = \text{Full Derivative}$
- Dirac action for Ψ:
  $S_D = \int d^4x \det(e)e^{\mu}_{A}\left(\Psi^{\dagger}(x)\gamma^{A}D_{\mu}\Psi(x) - \left(D_{\mu}\Psi(x)\right)^{\dagger}\gamma^{A}\Psi(x)\right) = S_{cosm}$!!!

$$S = \int d^4x\epsilon^{\mu\nu\alpha\beta}\epsilon_{ABCD}\left(\underbrace{\Lambda e^{A}_{\mu}e^{B}_{\nu}e^{C}_{\alpha}e^{D}_{\beta}}_{\sqrt{g}} - \underbrace{\mu_1\mathcal{F}^{AB}_{\mu\nu}e^{C}_{\alpha}e^{D}_{\beta}}_{\sqrt{g}R} + \underbrace{\ldots}_{\text{finite \# of terms}}\right)$$

How to check that quantum gravity reproduces classical one? Calculate DI correlators e.g. sphere surface

$$I_1(s) = \frac{\langle \int dx_1 \sqrt{g} \int dx_2 \sqrt{g} \delta(S(x_1, x_2) - s) \rangle}{\langle \int dx \sqrt{g} \rangle} (\simeq 2\pi^2 s^3)$$

But this can not be used for fermion $g_{\mu\nu}$. The classical metric tensor and classical effective action can be introduced by means of the Legendre transform [Wetterich 11, AV& Diakonov 12]

$$e^{W[\Theta]} = \int [D\phi^\dagger D\psi D\omega] \exp\left(S + \int \hat{g}_{\mu\nu}\Theta^{\mu\nu}\right), \qquad \hat{g}_{\mu\nu} \text{ built from fermions}$$

The classical metric:

$$g^{\rm cl}_{\mu\nu}(x) = \frac{\delta W[\Theta]}{\delta \Theta^{\mu\nu}(x)} \qquad \Rightarrow \qquad \Theta^{\mu\nu}[g^{\rm cl}]$$

Effective action is given by Legandre transform: $\Gamma[g^{\rm cl}] = W[\Theta] - g^{\rm cl}_{\mu\nu}\Theta^{\mu\nu}$. At the low energy limit it reads (due to diffeomorphism invariance):

$$S_{\rm low} = \int dx \sqrt{g^{\rm cl}} \left(c_1 + c_2 R(g^{\rm cl}) + ...\right),$$

$$\frac{c_1}{c_2^2} \simeq 5.8 \cdot 10^{-120} \qquad \text{How one can guarantee such tiny ratio?}$$

Lattice regularization of diffeomorphism invariant actions

$$Z[J,\lambda] = \int [D\omega\ D\psi^{\dagger} D\psi] e^{S},$$

$$S = \int d^4x \Big[\epsilon^{\mu\nu\rho\sigma} \epsilon_{ABCD} \left(\lambda_1 e^A_\mu e^B_\nu e^C_\rho e^D_\sigma + \lambda_2 e^A_\mu e^B_\nu e^C_\rho f^D_\sigma + \lambda_3 e^A_\mu e^B_\nu \mathcal{F}^{CD}_{\rho\sigma} + ... \right) + \lambda_4 \epsilon^{\mu\nu\rho\sigma} \mathcal{F}^{AB}_{\mu\nu} e^A_\rho e^B_\sigma + ... \Big]$$

In order to formulate quantum theory properly one has to regularize it at short distances. The most clear-cut regularization is lattice discretization

- Explicit invariance under gauge transformation (lattice gauge theory).
- In continuum limit, the lattice action reduces to diffeomorphism invariant (lattice diffeomorphism invariance).

Lattice regularization of diffeomorphism invariant actions

Lattice diffeomorphism invariance

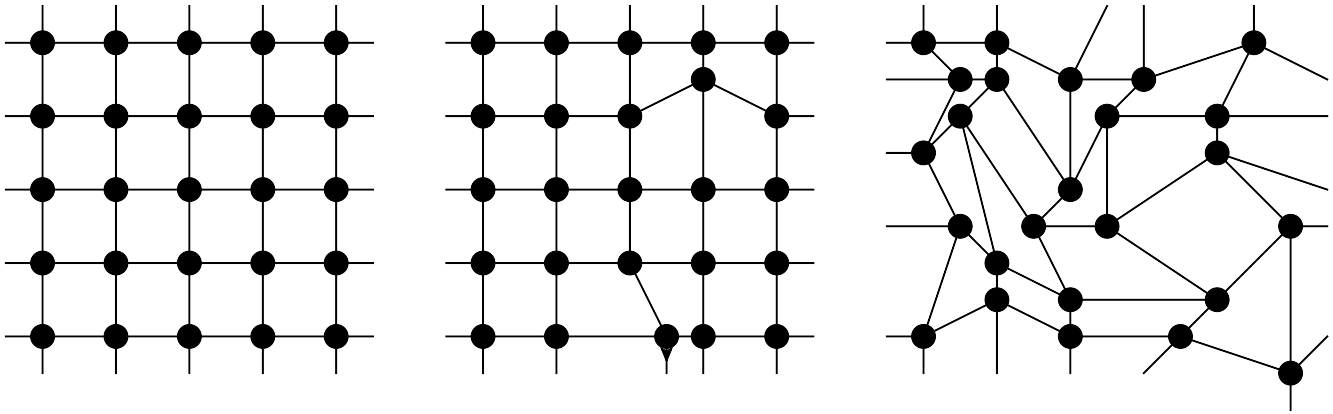
Spinor gravity is diffeomorphism invariant:

$$x \to x'(x) \qquad \int d^d x \mathcal{L}(x) = \int d^d x' \mathcal{L}(x'(x))$$

The lattice action should be independent on form of the lattice, only the neighborhood structure is important.

$$x(n) \to x'(x(n)) \qquad \sum_n \mathcal{L}(n|x) = \sum_n \mathcal{L}(n|x')$$

$x(n)$ is the map from discrete set of lattice vertices to the manifold.



Realization of the integral measure without diffeomorphism equivalent configurations

Explicit construction

Taking the simplex lattice with coordinates of vertices x_i^μ.
Volume of a simplex:

$$V_{\text{simplex}} = \frac{\epsilon^{i_0,i_1 i_2..i_d}}{(d+1)!}\frac{\epsilon^{\mu_1\mu_2..\mu_d}}{d!}(x_{i_1}^{\mu_1} - x_{i_0}^{\mu_1})..(x_{i_d}^{\mu_d} - x_{i_0}^{\mu_d})$$

Discredited version of a tetrad:

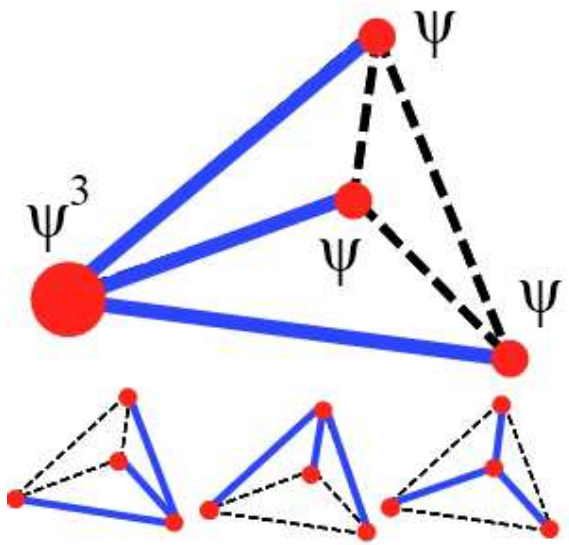
$$e_{ij}^A = \psi_i^\dagger \gamma^A U_{ij}\psi_j - \psi_j^\dagger U_{ji}\gamma^A\psi_i \simeq (x_j^\mu - x_i^\mu)e_\mu^A + \mathcal{O}(\Delta x^2)$$

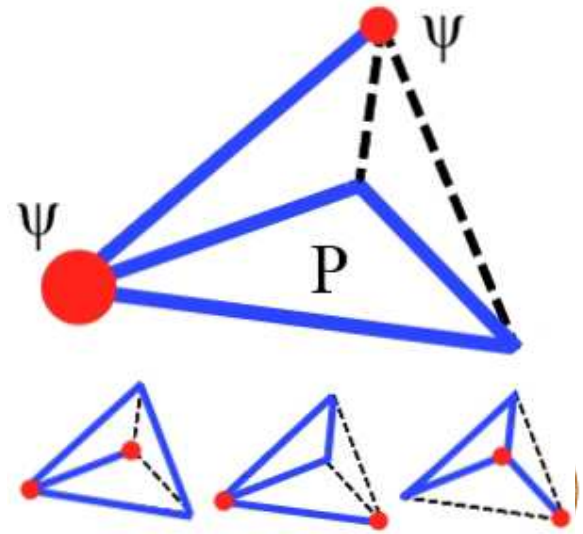
Discredited version of gauge strength tensor (plaquette)

$$P_{ijk}^{AB} = \frac{1}{d_f}\text{tr}\left(\sigma^{AB}U_{ij}U_{jk}U_{ki}\right)$$

U_{ij} is a holonomy (link):

$$U_{ij} = P\exp\left(\frac{-i}{2}\int_{x_i}^{x_j}\omega_\mu^{AB}\sigma^{AB}dx^\mu\right).$$

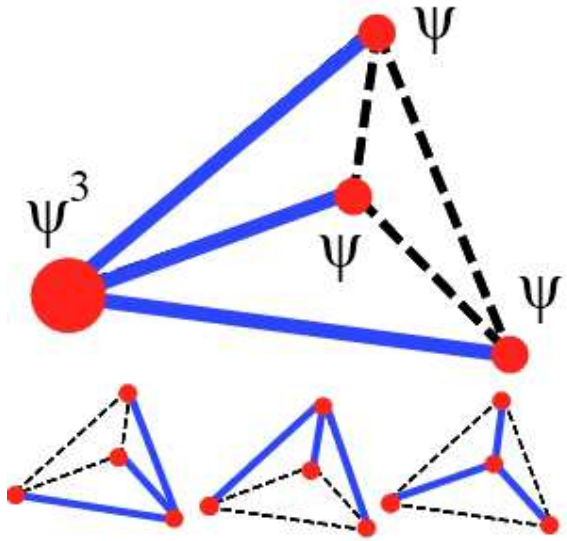




Cosmological term:

$$\sum_{\text{simplices}} \frac{\epsilon^{i_0 i_1 .. i_d}}{(d+1)!} \frac{\epsilon^{A_1 A_2 .. A_d}}{d!} e^{A_1}_{i_0 i_1} e^{A_2}_{i_0 i_2} ... e^{A_d}_{i_0 i_d}$$

$$\simeq \sum_{\text{simplices}} V_{\text{simplex}} \det(e) \left(1 + \mathcal{O}(\Delta x)\right) \simeq \int d^d x \det(e),$$

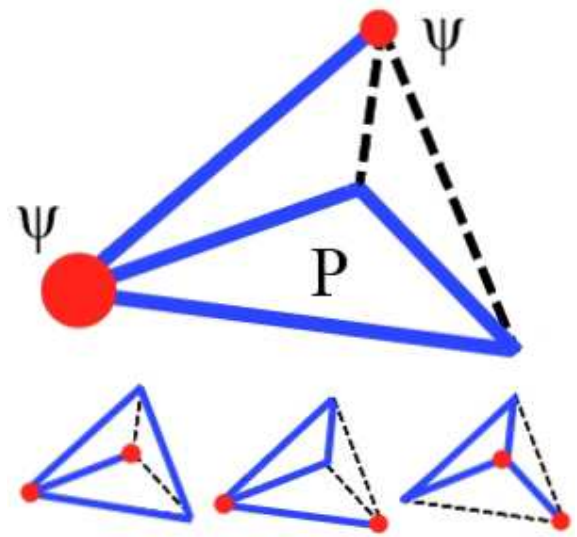


Einstein-Hilbert action:

$$4i \sum_{\text{simplices}} \frac{\epsilon^{i_0 i_1 .. i_d}}{(d+1)!} \frac{\epsilon^{A_1 A_2 .. A_d}}{d!} P^{A_1 A_2}_{i_0 i_1 i_2} e^{A_3}_{i_0 i_3} ... e^{A_d}_{i_0 i_d}$$

$$\simeq \int d^d x \epsilon^{A_1 .. A_d} \epsilon^{\mu_1 ... \mu_d} \mathcal{F}^{A_1 A_2}_{\mu_1 \mu_2} e^{A_3}_{\mu_3} ... e^{A_d}_{\mu_d}$$

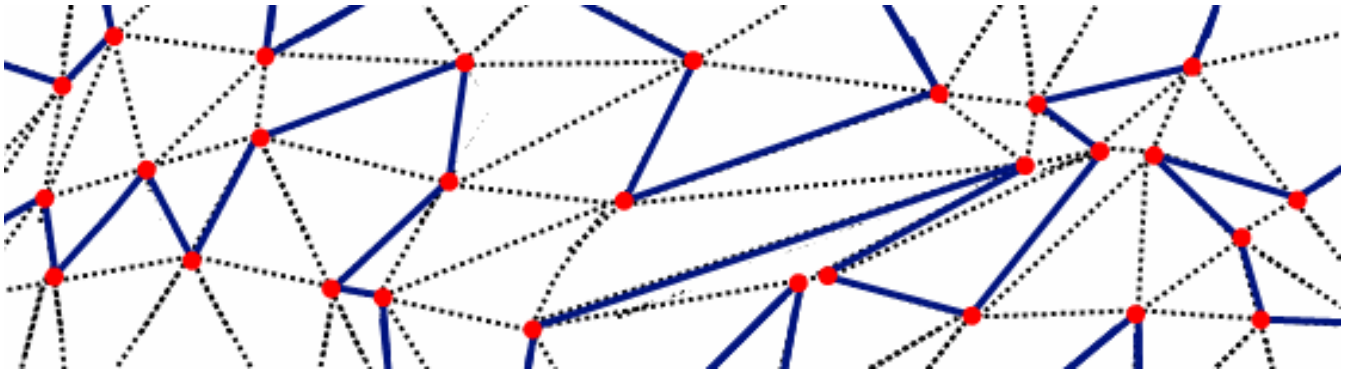
The lattice action is independent on the coordinate realization of the lattice.



Continuum limit

- In order to guarantee existence of continuum limit we need to search for long-range correlation. Usually (in lattice QCD) one uses the lattice RG to reveal the lattice-spacing dependance of parameters.
- This is a very unusual lattice field theory – with many-fermion vertices but no bilinear term for the fermion propagator! Typical (only?) behavior of correlations is fast (several lattice cells) exponential decay.
- There is no scale on diffeomorphism invariant lattice (by definition), and thus, no handle to turn the on the correlations (like $\beta \to \infty$ in QCD).
- The only stable way to provide long-range correlations is look for the phase transition and make fine-tuning. Exactly at the phase transition surface the correlation lengths are infinite and this automatically sets the constant Λ to be small.

Let us investigate such possibility on the simplest (but non-trivial) model: 2D spinor gravity.



2D spinor gravity

In 2D the Lorentz group $SO(2) \simeq U(1)$ and the spinor has two components ($\gamma^{1,2} = \sigma^{1,2}$, $\gamma^{\mathrm{FIVE}} = \sigma^3$).

$$\text{EH term} = \mathcal{F}^{AB}_{\mu\nu}\epsilon^{\mu\nu}\epsilon_{AB} = \text{Full Derivative}.$$

Only determinant-like terms are available

$$e^A_\mu = \psi^\dagger\gamma^A D_\mu\psi - (D_\mu\psi)^\dagger\gamma^A\psi, \qquad f^A_\mu = iD_\mu(\psi^\dagger\gamma^A\psi)$$

$$S = \int d^2x \left(\lambda_1 \det(f) + \lambda_2 \det(e) + \frac{\lambda_3}{2}\epsilon^{AB}\epsilon^{\mu\nu}e^A_\mu f^B_\nu\right)$$

$\Downarrow$ $\Downarrow$ Lattice regularization $\Downarrow$ $\Downarrow$

$$\tilde{S} = \sum_{\text{simplices}} \frac{\epsilon^{ijk}}{3!}\frac{\epsilon^{AB}}{2!}(\lambda_1\tilde{f}^A_{ij}\tilde{f}^B_{ik} + \lambda_2\tilde{e}^A_{ij}\tilde{e}^B_{ik} + \lambda_3\tilde{e}^A_{ij}\tilde{f}^B_{ik}),$$

$$\tilde{e}^A_{i,j} = \psi^\dagger_i\gamma^A U_{ij}\psi_j - \psi^\dagger_j U_{ji}\gamma^A\psi_i, \qquad \tilde{f}^A_{i,j} = i(\psi^\dagger_j U_{ji}\gamma^A U_{ij}\psi_j - \psi^\dagger_i\gamma^A\psi_i)$$

Exact results

Physical volume

Average physical volume is an extensive quantity (as it should be):

$$\langle V \rangle \Big|_{\lambda_1=0} = \langle \det(e) \rangle \Big|_{\lambda_1=0} = \frac{1}{Z} \frac{dZ}{d\lambda_2} \Big|_{\lambda_1=0} = \frac{M}{2\lambda_2}$$

Susceptibility of physical volume:

$$\langle \Delta V \rangle \Big|_{\lambda_1=0} = \langle V^2 \rangle - \langle V \rangle^2 \Big|_{\lambda_1=0} = -\frac{M}{2\lambda_2^2}$$

For large volumes the fluctuations dies out:

$$\sqrt{\Delta V^2}/V \sim 1/\sqrt{M} \to 0.$$

Scalar curvature

Average curvature and susceptibility of curvature:

$$\langle \det(e) R \rangle = 2\langle \mathcal{F}_{12}^{12} \rangle = 0, \qquad \langle (\det(e) R)^2 \rangle \sim M$$

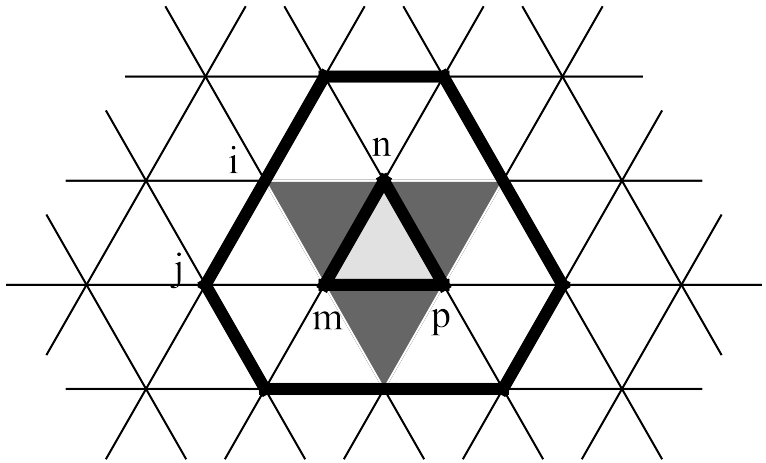
M is total number of simplices.

One may say that the model (in absence of sources) describes a flat background metric.

Dynamical mean field approach

- $Z = \int \prod_{i,j} d\psi_i^\dagger d\psi_j dU_{ij} e^S = \underbrace{\int d\psi_m^\dagger d\psi_m dU_{mn} e^{S_{mn}}}_{cavity} \underbrace{\int dU_{mi} e^{S_{mi}}}_{border} \underbrace{\int d\psi_i^\dagger d\psi_i dU_{ij} e^{S_{ij}}}_{enviroment}$
- $\int dU_{im} e^{S_{im}} = 1 + \sum_p O_p(\psi_m^\dagger, \psi_m, U_{mn}) O'_p(\psi_i^\dagger, \psi_i, U_{ij})$
- Cutting out one simplex out of infinite number changes nothing: $e^{S_{eff,mn}} = e^{S_{mn}} \left(1 + \sum_p O_p(\psi_m^\dagger, \psi_m, U_{mn})\right) \langle O'_p \rangle$
- Self-consistency equation:

$$\langle O_p \rangle = \frac{\int d\psi_m^\dagger d\psi_m dU_{mn} e^{S_{eff,mn}} O_p(\psi_m^\dagger, \psi_m, U_{mn})}{\int d\psi_m^\dagger d\psi_m dU_{mn} e^{S_{eff,mn}})}$$



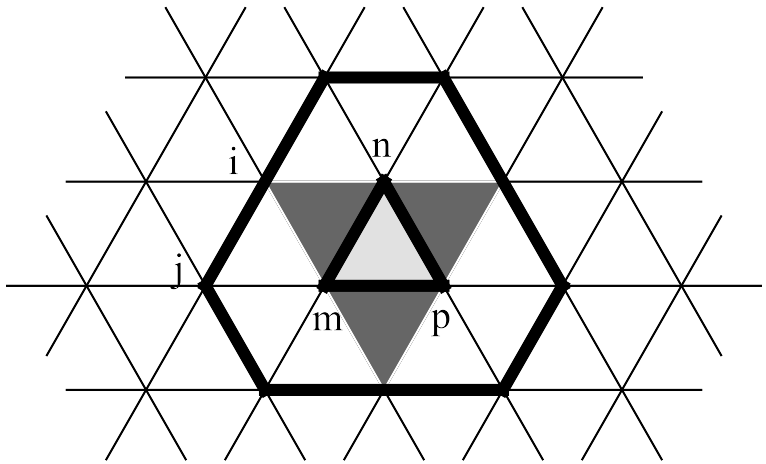
Dynamical mean field approach

Cavity method allows us to calculate only local quantities.
In 2D models there are 9 operators, and 9 equations of the form $\langle O_i \rangle = \frac{\mathrm{Pol}_{\sim 10}(\langle O \rangle)}{\mathrm{Pol}_{\sim 10}(\langle O \rangle)}$

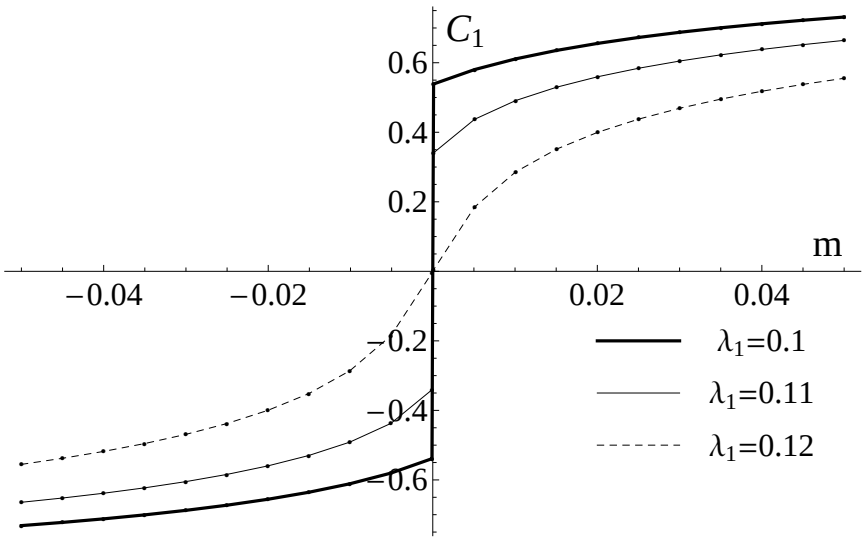
The accuracy of the method in 2D is about 15%.

$$\langle V \rangle \Big|_{\lambda_2 = 0} = 0.572 \frac{M}{\lambda_1} (\text{mean field}) \qquad \text{versus} \qquad 0.5 \frac{M}{\lambda_1} (\text{exact})$$

The accuracy can be increased by taking large cavities or greater dimensions. Approach is exact in $d \to \infty$.



Spontaneous chiral symmetry breaking



The model posses the chiral symmetry in additional to DI and local Lorentz symmetry.

$$\psi \to e^{\alpha\sigma^3}\psi, \qquad \psi^\dagger \to \psi^\dagger e^{\alpha\sigma^3}$$

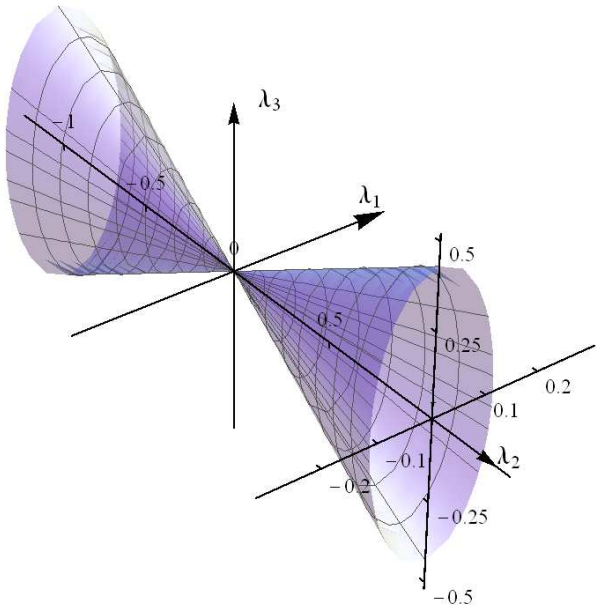
Looking for the spontaneous breaking of chiral symmetry we introduce an explicit symmetry breaking term:

$$S_{\chi-\mathrm{odd}} = \int d^2xi\,\det(e) m\psi^\dagger\psi,$$

then we consider chiral-odd operator at $m \to 0$.
In some sub-space of the parameters $(\lambda_1, \lambda_2, \lambda_3)$ the chiral condensate:

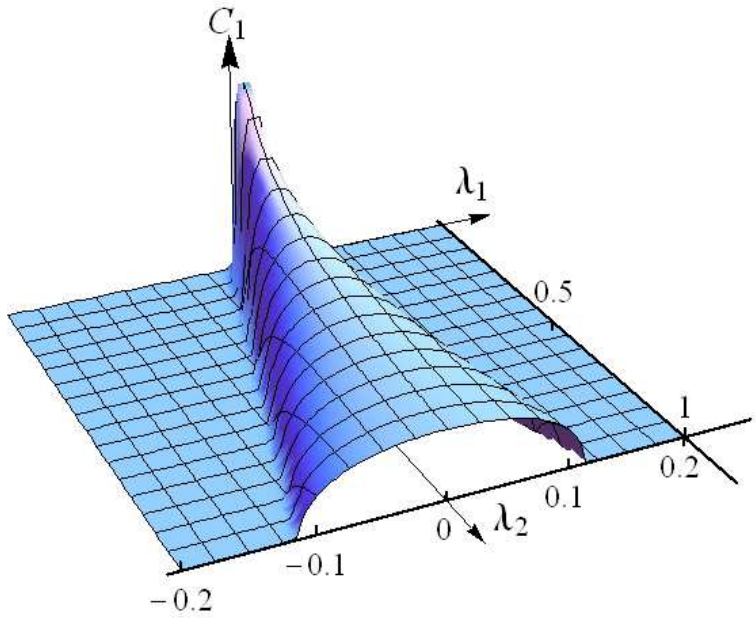
$$C_1 = i(\psi^\dagger\psi) \neq 0, \qquad \text{at } m = 0$$

Phase transition of 2^{nd} order



Spontaneous chiral symmetry breaking takes place inside a cone:

$$\lambda_2^2 < 77.23\lambda_1^2 + 5.36\lambda_3^2$$



The value of the chiral condensate $\langle C_1 \rangle$ at $\lambda_3 = 0$ plane. At phase transition line the condensate vanishes.

We have also checked for other possible symmetries breaking (e.g. fermion number violation) but did not anything in addition.

Effective Goldstone action

Under the chiral rotations the condensate operators transforms as:

$$C_1^{\pm} = i\left(\psi^\dagger\psi\right) \pm i\left(\psi^\dagger\sigma^3\psi\right) \longrightarrow e^{\pm i\alpha}C_1^{\pm}$$

Let the phase of condensate slowly vary from cell to cell:

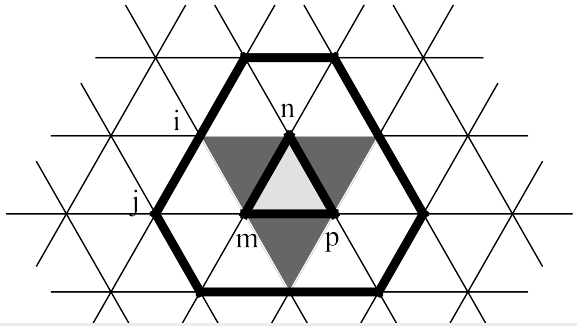
$$\langle C_1^{\pm}\rangle = \rho_1 e^{\pm i\alpha(\text{cell})}$$

Then using the same mean-field method we derive the low-energy action for the Goldstone-boson $\alpha(x)$:

$$e^{S_G(\alpha)} = \prod_{\text{cells}} \mathcal{Z}_{\text{cell}}(\alpha(\text{cell}))$$

$$\mathcal{S}_{GB} = -M\ln\mathcal{Z}(0) - \frac{M}{\mathcal{Z}}\left(\frac{\partial^2\mathcal{Z}}{\partial\alpha_i\partial\alpha_j} - \frac{1}{\mathcal{Z}}\frac{\partial\mathcal{Z}}{\partial\alpha_i}\frac{\partial\mathcal{Z}}{\partial\alpha_j}\right)\Bigg|_{\alpha=0}\Delta\alpha_i\Delta\alpha_j + \mathcal{O}(\Delta\alpha^4)$$

where M is a total number of cells and $\Delta\alpha_i$ is variation of α between cell i and current cell.



Effective Goldstone action

$$\Delta\alpha_i = \partial_\mu \alpha (x_i^\mu - x^\mu) + .., \qquad M = \sum_{\text{cell}} = \int \frac{d^2x}{V(\text{cell})}$$

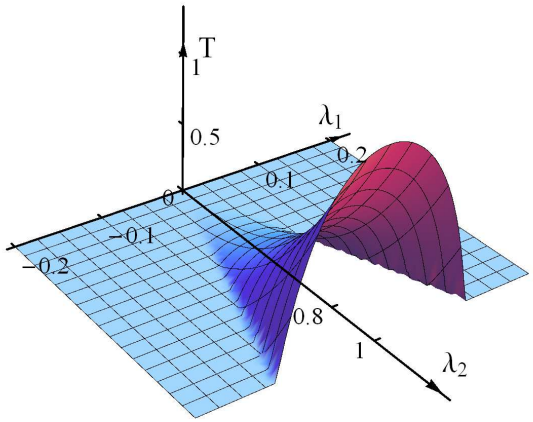
$$\lim_{\Delta x \to 0} \frac{1}{V(\text{cell})} \frac{1}{\mathcal{Z}_{10}} \frac{\partial^2 \mathcal{Z}_1}{\partial\alpha_i \partial\alpha_j} \Delta x_i^\mu \Delta x_j^\nu = -\sqrt{G}\, G^{\mu\nu}$$

where $G^{\mu\nu}(x)$ depends on $\lambda_{1,2,3}$, and can be calculated from self-consistency equation.

$$S_{GB} = \frac{1}{2} \int dx^2 \sqrt{G} G^{\mu\nu}(x) \partial_\mu \alpha \partial_\nu \alpha + \ldots$$

For a concrete map to the Cartazian coordinates we found:

$$\sqrt{G} G^{\mu\nu}(x)\Big|_{\text{regular lattice}} = T(\lambda_1, \lambda_2, \lambda_3)\delta^{\mu\nu}$$



"Gell-Mann-Oakes-Renner" relation

Explicit break of the chiral symmetry (e.g. by parameter m) leads to mass of Goldstone boson:

$$S_{GB} = \frac{1}{2}\int dx^2 \sqrt{G}(x)\left(G^{\mu\nu}(x)\partial_\mu\alpha\partial_\nu\alpha + \mu^2\alpha^2\right) + ..,$$

$$\mu^2 \sim m\, t(\lambda_{1,2,3})$$

At the phase transition surface μ goes to zero at non-zero m.

Mermin-Wagner theorem

A continuous symmetry cannot be spontaneously broken in $2d$ as the resulting Goldstone bosons would have an unacceptably large, actually divergent free energy. If, however, the Goldstone field $\alpha(x)$ is Abelian as here, the actual phase is, most likely, that of Berezinsky–Kosterlitz–Thouless where the chiral condensate $\rho\, e^{i\alpha}$ indeed vanishes owing to the violent fluctuations of $\alpha(x)$ defined on a circle $(0, 2\pi)$, but the correlation functions of the type $\langle e^{i\alpha(x)}\, e^{-i\alpha(y)}\rangle$ have a power-like behavior, and there is a phase transition depending on the original couplings of the theory.

Conclusion

- The lattice version of spinor gravity is constructed.
- The resulting theory is similar to the spin-foam models and to the strongly correlated fermion systems.
- The lattice mean-field approach is presented and applied to 2D spinor gravity.
- The presence of phase-transitions are shown, in particular chiral-phase transition.