# Identifying Codes in Graphs: a Special Class of Dominating Sets

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Identifying codes were introduced by Karpovsky, Chakrabarty and Levitin in 1998, and they can be applied, for example, to locating objects in sensor networks. Let a network be modelled by a simple, connected and undirected graph $G = (V, E)$ with vertex set $V$ and edge set $E$. We can place a sensor in any vertex $u$. A sensor is able to check its closed neighbourhood $N[u]$ (i.e., the adjacent vertices and itself) and report to a central controller if it detects something wrong there (for example, like a smoke detector). The idea is to place as few sensors as possible in such a way that we can uniquely determine where (that is, in which vertex) the problem occurs (if any) knowing only the set of sensors which gave us the alarm.

Let us denote the subset of vertices, where we placed the sensors, by $C$. In order to find the sought object (like fire in a building) in our network, we need to choose $C$ in the following way. Denote the set of sensors monitoring a vertex $u \in V$ by $I(u) = N[u] \cap C$. Suppose that $C$ satisfies the following two conditions: (i) $I(u) \neq \emptyset$ for every $u \in V$ and (ii) $I(u) \neq I(v)$ for all $u, v \in V$, $u \neq v$. Hence, $I(x)$ is the set of sensors giving the alarm if there is a problem in $x$, and since $I(v)$ is unique and nonempty for each $v \in V$, we can determine the vertex with a problem (if there is any). Such a subset $C \subseteq V$ satisfying the two requirements is called an *identifying code*. Obviously, the set $C$ is a dominating set of a graph if the first condition $I(u) \neq \emptyset$ is satisfied for all vertices $u$. It should be noticed that not all graphs admit an identifying code. Moreover, Slater has introduced a closely related concept of locating-dominating sets, where the second condition is replaced by $I(u) \neq I(v)$ where $u, v \in V \setminus C$. In the seminal paper, the identifying codes were generalized in two ways: 1) an $r$-*identifying code*: a sensor can check a closed neighourhood within distance $r$, 2) an $(r, \leq \ell)$-*identifying code*, which can uniquely locate several (up to $\ell$) objects in a network.

The original motivation for identifying codes came from finding malfunctioning processors in a multiprocessor system. The most studied underlying graphs include, for instance, square and triangular grids, hexagonal mesh, paths, cycles and binary hypercubes. More general graph theoretic questions have also been investigated over the years.

In this talk, we will consider recent developments in the field. In particular, conjectures concerning paths and cycles will be discussed, as well as optimal density of a 2-identifying code in the infinite hexagonal mesh.